A Novel Technique for Solving Multiobjective Fuzzy Linear Programming Problems

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Abstract–This **study considers multiobjective fuzzy linear programming (MFLP) problems in which the coefficients in the objective functions are triangular fuzzy numbers. The study proposing a new technique to transform MFLP problems into the equivalent single fuzzy linear programming problem and then solving it via linear ranking function using the simplex method, supported by numerical example.**

***Index Terms*—Triangular fuzzy numbers, Multiobjective fuzzy linear programming problems, Linear ranking function, Compromise solution.**

I. Introduction

A basic linear programming (LP) problem deals only with a single linear objective function subject to a linear constraint set, and the assumption that parameters are known with certainty. LP problems involving more than one possibly conflicting objective functions are called multiobjective linear programming (MLP) problems. Multiobjective fuzzy linear programming (MFLP) problems occur when the objective functions coefficients are fuzzy numbers (FNs).

Tanaka, et al. (1974a) first introduced fuzzy linear programming (FLP) problems, building on fuzzy environment presented by Bellman and Zadeh (1970). Zimmermann (1978) introduced the formulation of FLP problem and constructed a model of the problem also based on the fuzzy concepts of Bellman and Zadeh (1970). By the beginning of the current century, FLP problems have been used in broadly different real life problems (Iskander, 2002; Zhang, et al., 2005; Rong and Lahdelma, 2008; Chen and Ko, 2009; Peidro, et al., 2010; Hassanzadeh, et al., 2011).

Ebrahimnejad and Tavana (2014) classified FLP problems into five main groups based on findings of various researchers (Zimmermann, 1987; Luhandjula, 1989; Inuiguchi, et al., 1990; Buckley and Feuring, 2000; Hashemi, et al., 2006; Dehghan, et al., 2006; Allahviranloo, et al., 2008; Hosseinzadeh Lotfi, et al., 2009; Kumar, et al., 2011).

In a fully fuzzified LP problem where all the parameters and variables are FNs, Buckley, and Feuring (2000) changed the problem of maximizing an FN, the objective function's value into an MFLP problems. They proved that all undominated set to MFLP problems can be explored by fuzzy flexible programming.

An interactive fuzzy programming was proposed by Sakawa, et al. (2000) to solve MLP problems with fuzzy parameters. After defuzzifying the fuzzy goals of the decision makers (DMs), a satisfactory solution is derived efficiently by updating the satisfactory degrees of the DMs at the topmost levels with respectfulness stable satisfactory among all levels.

MFLP vector optimization problems of a fuzzy nature were considered by Cadenas and Verdegay (2000) who assumed that all the objective functions involved come from the same DM with FN coefficients and they can be defined by different DMs.

Stanciulescu, et al. (2003) formulated a multiobjective decision-making process in which the coefficients of the objective functions and the constraints are fuzzy as MFLP problems. Their method uses fuzzy decision variables with a joint membership function instead of crisp decision variables. The lower bound fuzzy decision variables set up the lower bounds of the decision variables and generalize to lower-upper bound fuzzy decision variables that in turn set up the upper bounds of the decision variables too. The Optimal solutions (OSs) of the problem and their method supply to the DM regions containing potential satisfactory solutions around the OSs.

Cadenas and Verdegay (2000) used a ranking function in dealing with MFLP problems, multiobjective mathematical programming problems, vector optimization programming (VOP) problems, and Fuzzy Multiobjective Optimization problems. Ganesan and Veeramani (2006) introduced FLP with symmetric trapezoidal fuzzy numbers and proposed to solve this kind of problems using ranking function for FNs, without converting the problem to crisp LP problem. In the study of MFLP model for supplier selection in supply chain (Amid, et al., 2006), an MFLP model was developed with vagueness, imprecision of the goals, constraints, and parameters in which the decision-making has been made difficult for such kind of problems (Mahdavi-Amiri and Nasseri, 2006).

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Wu (2008a) derived the optimality conditions for LP problems with fuzzy coefficients when considering the orderings of the set of all FNs and proposed two solution approaches. Nondominated solution was proposed in the MLP problem by naturally eliciting the optimality conditions. To solve MFLP problems, Wu (2008b) converted the problem into a VOP problem by employing the embedding proposition and using appropriate linear defuzzification functions.

In some MFLP models, both the objective functions and the constraints are fuzzy. Furthermore, the coefficients of the decision variables in the objective functions, constraints, and the right-hand sides of the constraints are assumed to be FNs with either triangular or trapezoidal membership functions. Iskander (2008) proposed to utilize possibilistic programming to transform such MFLP problems as previously modeled (Negi and Lee, 1993) into its equivalent crisp programming according to the author's modifications. Iskander (2002, 2008) used two main criteria with the same evaluation concept in MFLP: The global criterion method and the distance functions method.

Baky (2009) proposed fuzzy goal programming algorithm for solving decentralized MFLP problems in the form of bilevel programming problems to obtaining OS for the problem. In another paper, the researcher (Baky, 2010) presented two new algorithms to solve MFLP problems through the fuzzy goal programming approach.

Amid, et al. (2011) developed a weighted max–min method and used it to solve MFLP problems to help managers of supplier selection and allow them to assign the order quantities to each supplier based on supply chain strategies.

Gupta and Kumar (2012) studied Chiang's method (Chiang, 2005) and pointed out the shortcomings in the latter's method. Hence, they proposed a new method to overcome these weaknesses of the MFLP problems by representing all the parameters in the system as (λ, ρ) interval-valued FNs.

In their review paper, Hamadameen and Zainuddin (2013) focused on various kinds of MFLP problems. They discussed the main studies in the recent years comprehensively. They considered problems with fuzziness in both the objective functions and constraints and analyzed MFLP problems chronologically. They also described problem formulation and the various research methodologies in MFLP problems. In addition, they surveyed many transformation methods that have been used to convert MFLP problems into their corresponding equivalent deterministic MLP problems. Moreover, they also addressed OSs for the original problem in each study.

Luhandjula and Rangoaga (2014) presented a new approach in solving continuous optimization problems based on the nearest interval approximation operator for dealing with an MFLP problem. They established a Karush-Kuhn-Tucker (KKT) kind of pareto optimality conditions. There were two crucial algorithms in the proposed method; the first gave nearest interval approximation to a given FN, and the second provided KKT conditions to deliver a pareto OS.

In this study, we address the MFLP problems in which objective functions' coefficients are triangular fuzzy numbers (T_rFNs). The study utilizes a linear ranking function through simplex method, in addition a new method to transform the MFLP problems into single FLP problem and find a compromise solution for the original problem, in which consists in minimizing the sum of distances from the objective functions to predefined ideal values. This paper is organized as follows: Section 2 defines fuzzy concepts and algebra properties of T_rFNs. Section 3 addresses linear ranking functions and the comparison of FNs. In addition, it gives the mathematical formulation of the T_rFNs. Section 4 defines the mathematical formulation for FLP problem and MFLP problems. Section 5 addresses OS, simplex method, and compromise solution for MFLP problems. Solution algorithms are presented in Section 6. In Section 7, to illustrate the proposed method, a numerical example is solved. Conclusions are discussed in Section 8.

II. Preliminaries of Fuzzy Concepts

This study uses some of the concepts of fuzzy sets. We list here some definitions and properties.

A. Basic Definitions

Fuzzy set: Let X be the universal set. $\tilde{A}$ is called a fuzzy set in X if $\tilde{A}$ is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$; where $\mu_{\tilde{A}}(x)$ is the membership function of $x \in \tilde{A}$ (Sakawa, 1993). Note that the membership function of $\tilde{A}$ is a characteristic (indicator) function for $\tilde{A}$ and it shows to what degree $x \in \tilde{A}$.

α-level set: The α-level set of $\tilde{A}$ is the set $\tilde{A}_\alpha = \{x \in R | \mu_{\tilde{A}}(x) \geq \alpha\}$, where $\alpha \in [0,1]$. The lower and upper bounds of α-level set $\tilde{A}$ are finite numbers represented by inf $x \in \tilde{A}_\alpha$ and sup $x \in \tilde{A}_\alpha$, respectively (Wang, 1997; Sakawa, 1993; Yager and Filev, 1994).

Normal Fuzzy Set: The height of a fuzzy set is the largest membership value attained by any point. If the height of fuzzy set equals one, it is called a normal fuzzy set (Wang, 1997).

The core (modal): The core of a fuzzy set $\tilde{A}$ of X is the crisp subset of X consisting of all elements with membership grade one, or core $(\tilde{A}) = \left\{x | \tilde{A}(x) = 1 \text{ and } x \in \mathrm{X}\right\}$ (Yager and File, 199).

The support: The support of a fuzzy set $\tilde{A}$ is a set of elements in X for which $\tilde{A}(x)$ is positive, that is, supp $\tilde{A} = \{x \in X | \mu_{\tilde{A}}(x) > 0\}$ (Wang, 1997; Sakawa, 1993).

Fuzzy convex set: A fuzzy set $\tilde{A}$ is convex if $\mu_{\tilde{A}}\left(\lambda x + (1-\lambda) y\right) \geq min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}, \forall\, x, y \in X \wedge \lambda \in [0,1]$ (Wang, 1997; Sakawa, 1993).

Convexity and fuzzy number (FN): A convex fuzzy set $\tilde{A}$ on $\Re$ is an FN if: one its membership function is piecewise continuous; two there exist three intervals $[a, b]$, $[b, c]$ and $[c, d]$ such that $\tilde{A}$ is increasing on $[a, b]$, equal to 1 on $[b, c]$, decreasing on $[c, d]$, and equal to 0 elsewhere $\forall\, a, b, c \in \mathbb{R}$ (Mahdavi-Amiri and Nasseri, 2006; Mahdavi-Amiri and Nasseri, 2007).

The trapezoidal fuzzy number (T_pFN): Let $\tilde{A} = (a^L, a^U, \alpha, \beta)$ be the T_pFN, where $[a^L, a^U]$ is the modal set of $\tilde{A}$, and

$[a^L - \alpha,\ a^U + \beta]$ its support part (Mahdavi-Amiri and Nasseri, 2006; Mahdavi-Amiri and Nasseri, 2007) (Fig. 1).

If $a = a^L = a^U \in \tilde{A}$ then the T_pFN is reduced to T_rFN and denoted by $\tilde{A} = (a, \alpha, \beta)$ (Fig. 2). Thus, $\tilde{A} = (a, \alpha, \beta) \subset (a^L, a^U, \alpha, \beta)$. Since the study is focused on MFLP problems with T_rFNs, the next section lists algebra properties specific to such FNs.

B. Algebra Properties of FNs

Let $\tilde{A}_1, \tilde{A}_2 \in$ TrFNs, such that $\tilde{A}_1 = (a_1, \alpha_1, \beta_1)$ and $\tilde{A}_2 = (a_2, \alpha_2, \beta_2)$, then based on Zadeh (1965), Dubois and Prade (1978), and Sakawa (1993) the following rules apply:

1. Addition: $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1, \alpha_1, \beta_1) \oplus (a_2, \alpha_2, \beta_2) = (a_1 + a_2, a_1 + \alpha_1, \beta_1 + \beta_2)$
2. Image $\tilde{A}_1$ = Image $(a_1, \alpha_1, \beta_1) = -\tilde{A}_1 = -(a_1, \alpha_1, \beta_1) = (-a_1, \beta_1, \alpha_1)$
3. Subtraction: $\tilde{A}_1 \ominus \tilde{A}_2 = (a_1, \alpha_1, \beta_1) \ominus (a_2, \alpha_2, \beta_2) = (a_1, \alpha_1, \beta_1) \oplus (-a_2, \beta_2, \alpha_2) = (a_1 - a_2, \alpha_1 + \beta_2, \beta_{1+} \alpha_2)$
4. Multiplication: $\tilde{A}_1 \otimes \tilde{A}_2 =$
$$\left(a_1,\alpha_1,\beta_1\right)\otimes\left(a_2,\alpha_2,\beta_2\right) \cong \begin{cases} \left(a_1a_2,\ a_1\alpha_2 + a_2\alpha_1,\ a_1\beta_2 + a_2\beta_1\right); a_1 < \tilde{0}, a_2 > \tilde{0} \\ \left(a_1a_2,\ a_2\alpha_1 - a_1\beta_2,\ a_2\beta_1 - a_1\alpha_2\right); a_1 > \tilde{0}, a_2 < \tilde{0} \\ \left(a_1a_2, -a_2\beta_1 - a_1\beta_2, -a_2\alpha_1 - a_1\alpha_2\right); a_1 < \tilde{0}, a_2 < \tilde{0} \end{cases}$$
5. Scalar multiplication:
$$\delta \otimes \tilde{A}_1 = \delta \otimes \left(a_1, \alpha_1, \beta_1\right) = \begin{cases} \left(\delta a_1, \delta\alpha_1, \delta\beta_1\right);\ \delta > 0 \\ \left(\delta a_1, -\delta\beta_1, -\delta\alpha_1\right);\ \delta < 0 \end{cases}$$
6. Inverse: $\tilde{A}_1^{-1} = \left(a_1, \alpha_1, \beta_1\right)^{-1} = \left(a_1^{-1},\ \beta_1 a_1^{-2}, \alpha_1 a_1^{-2}\right)$
7. Division: $$\frac{\tilde{A}_1}{\tilde{A}_2} = \frac{\tilde{A}_1}{\tilde{A}_2^{-1}} = \frac{a_1, \alpha_1, \beta_1}{a_2, \alpha_2, \beta_2} = \left(\frac{a_1}{a_2}, \frac{\beta_2 a + \alpha_1 a_2}{a_2^2}, \frac{\alpha_2 a_1 + \beta_1 a_2}{a_2^2}\right); \forall \tilde{A}_1, \tilde{A}_2 > \tilde{0}$$.

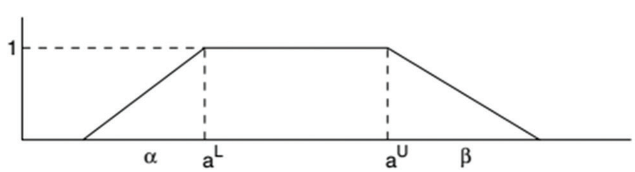

Fig. 1 Trapezoidal fuzzy number

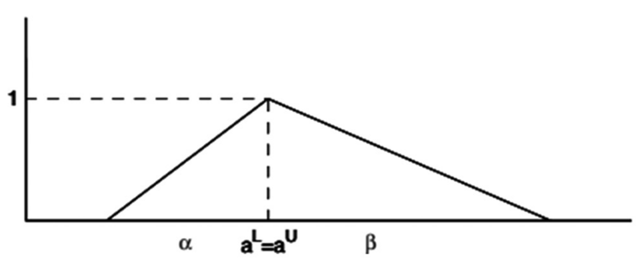

Fig. 2 Trinomial fuzzy number

Note that similar formulas hold when $\tilde{A}_1 < \tilde{0}, \tilde{A}_2 > \tilde{0}$ or $\tilde{A}_1 > \tilde{0}, \tilde{A}_2 < \tilde{0}$

8. $\tilde{A}_1 = \tilde{0} \Leftrightarrow \tilde{A}_1 = \left(0, 0, 0\right)$.

III. Ranking Functions and the Comparison of *FN*

The first step in solving MFLP is to defuzzify the fuzzy assertion. One of the several tools used to achieve this aim is a ranking function (Fang and Hu, 1996; Lai and Hwaang, 1992; Maleki,et al., 2000; Shoacheng, 1994; Tanaka, et al., 1974b; Maleki, 2003; Mahdavi-Amiri and Nasseri, 2006; Mahdavi-Amiri and Nasseri, 2007; Ebrahimnejad, 2011; Ullah Khan,et al., 2013) based on the comparison of the FNs (Wang and Kerre, 2001; Garcia-Aguado and Verdegay, 1993; Maleki, 2003).

This study focuses on a ranking function by Mahdavi-Amiri and Nasseri (2007). This ranking function is particularly suitable for T_pFNs. It transforms the FN to a real number. A ranking function $\Re : F\left(R\right) \to \mathbb{R}$ is a map which transforms each FN into its corresponding real line, where a natural order exists (Roubens and Jacques, 1991; Fortemps and Roubens, 1996; Mahdavi-Amiri and Nasseri, 2007; Nasseri, et al., 2005).

For an T_pFN $(\tilde{a}) = (a^L, a^U, \alpha, \beta)$, Yager (1981) proposed the special kind of $\Re\left(\tilde{a}\right)$ formulated as follows:

$$\Re\left(\tilde{a}\right) = \frac{1}{2}\int_0^1 \left(inf\tilde{a}_\lambda + sup\tilde{a}_\lambda\right) d\lambda = \frac{a^L + a^U}{2} + \frac{\beta - \alpha}{4} \tag{1}$$

Based on the definition of TrFN, and TpFN, (1) can convert into the following form for the TrFNs:

$$\Re\left(\tilde{a}\right) = a + \frac{\beta - \alpha}{4} \tag{2}$$

In this study, we focus only on the linear ranking function. We list some of its properties on FNs.

For all $\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4 \in$ FNs, and $\delta \in \Re$ then:

i. $\tilde{a}_1 \geq_\Re \tilde{a}_2 \Leftrightarrow \Re\left(\tilde{a}_1\right) \geq \Re\left(\tilde{a}_2\right)$
ii. $\tilde{a}_1 >_\Re \tilde{a}_2 \Leftrightarrow \Re\left(\tilde{a}_1\right) > R\left(\tilde{a}_2\right)$
iii. $\tilde{a}_1 =_\Re \tilde{a}_2 \Leftrightarrow \Re\left(\tilde{a}_1\right) = \Re\left(\tilde{a}_2\right)$
iv. $\tilde{a}_1 \leq_\Re \tilde{a}_2 \Leftrightarrow \Re\left(\tilde{a}_1\right) \leq \Re\left(\tilde{a}_2\right)$
v. $\delta\tilde{a}_1 +_\Re \tilde{a}_2 = \delta\Re\tilde{a}_1 + \Re\left(\tilde{a}_2\right)$
vi. $\tilde{a}_1 =_\Re \tilde{0} \Leftrightarrow \Re\left(\tilde{a}_1\right) = \Re\left(\tilde{0}\right) = 0$
vii. $\tilde{a}_1 \geq_\Re \tilde{a}_2 \Leftrightarrow \tilde{a}_1 - \tilde{a}_2 \geq_\Re 0 \Leftrightarrow -\tilde{a}_2 \geq_\Re -\tilde{a}_1$
viii. $\tilde{a}_1 \geq_\Re \tilde{a}_2 \wedge \tilde{a}_3 \geq_\Re \tilde{a}_4 \Leftrightarrow \tilde{a}_1 + \tilde{a}_3 \geq_\Re \tilde{a}_2 + \tilde{a}_4$

IV. Problem Formulation

A. FLP Problem

The mathematical formulation of the FLP problem can be written as follows:

$$Max\ \tilde{z}(x) =_{\mathfrak{R}} \tilde{c}x$$
$$s.t. Ax \le b \quad (3)$$
$$x \ge 0$$

Where $\tilde{c}^T$, A and b are of dimentions (n, 1), (m, n) and (m, 1), respectively. A feasible solution for (3) is the vector $x \in R^n$ which satisfies the constraints and their signs. In addition, x^* is an optimal feasible solution for (3) if and only if $\tilde{c}x^* \ge_{\mathfrak{R}} \tilde{c}x$, for all feasible solution x. On the other side of the constraints of (3), if Rank (A, b) = Rank (A) = m, then after partition and rearranging of the columns of $A = [B, N]$, where the nonsingular matrix $B = (m, m)$, and rank (B) = m where $x_B = B^{-1} b$, $x_N = 0$; then, the basic solution point $x\left(x_{B^T}, x_{N^T}\right)^T$ is called the basic feasible solution (BFS) for the system in (3), where *b*and *n*are basic matrix, and non-basic matrix, respectively (Nasseri, et al., 2005).

B. MFLP Problem

The mathematical formulation of the MFLP problems can be written as follows:

$$Max\ \tilde{Z}_i(x) =_{\mathfrak{R}} \sum_{j=1}^{n} \tilde{c}_{ij}\ x_j; i = 1,\ldots,r$$
$$Min\ \tilde{Z}_i(x) =_{\mathfrak{R}} \sum_{j=1}^{n} \tilde{c}_{ij} x_j; i = r+1,\ldots,s$$
$$s.t. Ax \le b \quad (4)$$
$$x \ge 0$$

Where $\tilde{c}^T$, A, b and x are as defined in (3).

V. Simplex Method and Feasible Solution for FLP Problem

For the FLP problem in (3), after converting it to the standard form:

$$Max \tilde{z}(x) =_{\mathfrak{R}} \tilde{c}_B x + \tilde{c}_N s$$
$$s.t. B x_B + N s_N = b \quad (5)$$
$$x_B, s_N \ge 0$$

Since,

$$x_B + B^{-1} N s_N = B^{-1} b,\ \tilde{z} + \left(\tilde{c}_B B^{-1} N - \tilde{c}_N\right) s_N =_{\mathfrak{R}} \tilde{c} B^{-1} b.$$

initially $s_N = 0$ thus, $x_B = B^{-1} b$, $\tilde{z} =_{\mathfrak{R}} \tilde{c}_B B^{-1} b$. We can express (3) in Table I.

In Table I, we have (Dantzig, 1963; Maleki, et al., 2000; Nasseri, et al., 2005; Mahdavi-Amiri and Nasseri, 2007; Sharma, 2012):

1. The fuzzy objective row, $\tilde{\gamma}_j =_{\mathfrak{R}} \left(\tilde{c}_B B^{-1} a_j - \tilde{c}_j\right)_{j \ne B_j}$ continuous the $\tilde{\gamma}_j =_{\mathfrak{R}} \tilde{z}_j - \tilde{c}_j$ for the nonbasic variables.
2. For the feasible OS, it should be $\tilde{\gamma}_j =_{\mathfrak{R}} \ge 0, \forall j \ne B_i$.
3. If, $\tilde{\gamma}_k <_{\mathfrak{R}} 0, \forall k \ne B_i$, then exchange x_{Br} by x_k. After that satisfying $\gamma_k =_{\mathfrak{R}} B^{-1} a_k$.
4. If, $\tilde{\gamma}_k \le_{\mathfrak{R}} 0$, then x_k is an unbounded solution for the problem.

TABLE I
The FLP Problem

	Objective function value: $\tilde{z}$	RHS	Basic variable: x_B	Nonbasic variable: S_N
x_B	0	1	$B^{-1} b$	$B^{-1} b$
$\tilde{z}$	1	$\tilde{c}_B B^{-1} b$	$\tilde{0}$	$\tilde{c}_B B^{-1} N - \tilde{c} N$

RHS: Right hand side, FLP: Fuzzy linear programming

5. If an m exist such that $\tilde{z}_m - \tilde{c}_m <_{\mathfrak{R}} \tilde{0}$ and there exist a basic index i in which $y_{im} > 0$, then a pivoting row p can be found in which the pivoting y_{pm} yields a feasible tableau corresponding fuzzy objective value.
6. For any feasible solution to FLP problem, there are some columns not in the basic solution in which $\tilde{z}_m - \tilde{c}_m <_{\mathfrak{R}} \tilde{0}$ and $y_{im} \le 0, i = 1,\ldots,s$ then the problem is unbounded.

VI. Compromise Solution

Since many objectives of the system usually conflict with each other, an improvement of one objective may mean the sacrifice in another. A compromise solution lets the DMs specify partial preferences among conflicting objectives so that there will be less alternative solutions. This can mean making adjustments to others.

A. A Compromise Solution for MFLP Problems

Since there may be conflicts among the multiple objectives in the MFLP problems in (4) under the same set of constraints, it is difficult to find a solution which satisfies all of those objective functions. Thus, a compromise solution is most realistic and practical for such kinds of the problems. The decision variable may not be common to all OSs in the presence of conflicts among objectives. However, the common set of decision variables between objective functions is necessary to facilitate selection of the best compromise solution. The next section summarized the solution algorithm for the method used in this study.

B. Solution Algorithms

Let us now describe the algorithm step by step:

Step 1: Consider the problem as the mathematical form in (4).

Step 2: Convert (4) into the standard form as:

$$Max\ \tilde{Z}_i(x) =_{\mathfrak{R}} \sum_{j=1}^{n} (\tilde{c}_{ijB} x_j + \tilde{c}_{ijN} s_j); i = 1,\ldots,r$$
$$Min\ \tilde{Z}_i(x) =_{\mathfrak{R}} \sum_{j=1}^{n} (\tilde{c}_{ijB} x_j + \tilde{c}_{ijN} s_j); i = r+1,\ldots,s$$
$$s.t. B x_B + N s_N \le b$$
$$x_B, s_N \ge 0 \quad (6)$$

Step 3: Use the tableau notifications in Table I to solve each FLP problem in the form of (5) by simplex method.

Step 4: Assign $\tilde{v}_i$ to the optimum value of the objective function $\tilde{Z}_i; i = 1,\ldots,r,\ldots,s$.

Step 5: Convert (6) into its corresponding FLP problem as follows:

$$Max\ \tilde{Z}(x) =_{\Re} \sum_{j=1}^{n} [Max\ \tilde{c}_{ij} x_j \oslash \tilde{v}_i]_{i=1,\ldots,r}$$

$$-\sum_{j=1}^{n} [Max\ \tilde{c}_{ij} x_j \oslash \tilde{v}_i]_{i=r+1,\ldots s} : \forall \tilde{v}_i \neq \tilde{0}$$

$$s.t. Bx_B + Ns_N \leq b$$
$$x_B,\ s_N \geq 0 \tag{7}$$

Step 6: Find an OS for (7) which will give the compromise solution for the original problem in the (4).

VII. Numerical Example

Consider the following MFLP problems:

$$Max\ \tilde{Z}_1(x) =_{\Re} (5,2,5)x_1 + (6,3,6)x_2 + (5,3,7)x_3$$
$$Max\ \tilde{Z}_2(x) =_{\Re} (4,7,11)x_1 + (5,5,9)x_2 + (3,6,10)x_3$$
$$Min\ \tilde{Z}_3(x) =_{\Re} (-1,3,2)x_1 + (-3,4,1)x_2 + \left(\frac{-5}{2},9,7\right)x_3$$
$$Min\ \tilde{Z}_4(x) =_{\Re} (-3,4,4)x_1 + (-3,4,6)x_2 + (-3,4,8)x_3$$
$$s.t. 3x_1 - x_2 + 3x_3 \leq 7$$
$$-2x_1 + 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_j \geq 0,\ j = 1,2,3 \tag{8}$$

Solution: First, we solve each objective function subject to the constraints individually, as:

$$Max\ \tilde{Z}_1(x) =_{\Re} (5,2,5)x_1 + (6,3,6)x_2 + (5,3,7)x_3 + \tilde{0}\sum_{j=1}^{3} s_j$$
$$s.t. 3x_1 - x_2 + 3x_3 + s_1 = 7$$
$$-2x_1 + 4x_2 + s_2 = 12$$
$$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$$
$$x_j, x_j \geq 0,\ j = 1,2,3 \tag{9}$$

From Table II, we have $\left\{\left(\tilde{z}_j - \tilde{c}_j\right)\right\}$

$$=_{\Re} \left\{(-5,5,2),(-6,6,3),(-5,7,3),\tilde{0},\tilde{0},\tilde{0}\right\};\ j = 1,\ldots,6.$$

j = 1.,6

Since, $\left\{\gamma_j\right\} = \left\{\Re\left(\tilde{\gamma}_j\right)\right\} = \left\{-5\frac{3}{4}, -6\frac{3}{4}, -6, 0, 0, 0\right\}$,

j = 1.,6, thus, x_2 should enter the basic solution, and the leaving variable s_2.

The result is as shown in Table III.

From Table III, we have

$$\left\{\left(\tilde{z}_j - \tilde{c}_j\right)\right\} =_{\Re} \left\{\left(-8,8,\frac{7}{2}\right),\tilde{0},(-5,7,3),\tilde{0},\left(\frac{3}{2},\frac{3}{4},\frac{3}{2}\right),\tilde{0}\right\};$$

j = 1.,6

TABLE II
The Status of the Solution-1

B	$\tilde{ci}$	RRHS	(5,2,5) x_1	(6,3,6) x_2	(5,3,7) x_3	$\tilde{0}$ s_1	$\tilde{0}$ s_2	$\tilde{0}$ s_1	Min ratio
s_1	$\tilde{0}$	7	3	−1	3	1	0	0	_
s_2	$\tilde{0}$	12	−2	4	0	0	1	0	3←
s_3	$\tilde{0}$	10	−4	3	8	0	0	1	$3\frac{1}{3}$
	$\tilde{z}$	$\tilde{0}$	(−5,5,2)	(−6,6,3) ↑	(−5,7,3)	$\tilde{0}$	$\tilde{0}$↓	$\tilde{0}$	$\Re$

Table III
The Status of the Solution-2

B	$\tilde{ci}$	RRHS	(5,2,7) x_1	(6,3,6) x_2	(5,3,7) x_3	$\tilde{0}$ s_1	$\tilde{0}$ s_2	$\tilde{0}$ s_1	Min ratio
s_1	$\tilde{0}$	10	3	0	3	1	$\frac{1}{4}$	0	4←
s_2	(6, 3, 6)	3	−2	1	0	0	$\frac{1}{4}$	0	-
s_3	$\tilde{0}$	1	−4	0	8	0	$\frac{-3}{4}$	1	-
	$\tilde{z}$(18,9,18)		$\left(-8,8,\frac{7}{2}\right)$↑	$\tilde{0}$	(-5,7,3)	$\tilde{0}$↓	$\left(\frac{3}{2},\frac{3}{4},\frac{3}{2}\right)$	$\tilde{0}$	$\Re$

Since, $\left\{\gamma_j\right\} = \left\{\Re\left(\tilde{\gamma}_j\right)\right\} = \left\{-9\frac{1}{8}, 0, -6, 0, 1\frac{11}{16}, 0\right\},\ j = 1,\ldots,6$.

Thus, x_1 should enter the basic solution and the leaving variable s_1. The result is as shown in Table IV.

Now,

$$\left\{\left(\tilde{z}_j - \tilde{c}_j\right)\right\} =_{\Re} \left\{\begin{matrix}\tilde{0},\tilde{0},(-5,7,3),\tilde{0},\left(\frac{23}{5},\frac{56}{5},\frac{63}{5}\right), \\ \left(\frac{16}{3},\frac{7}{3},\frac{16}{3}\right),\left(\frac{23}{10},\frac{11}{10},\frac{23}{10}\right),\tilde{0}\end{matrix}\right\};\ j = 1,\ldots,6,$$

and

$$\left\{\gamma_j\right\} = \left\{\Re\left(\tilde{\gamma}_j\right)\right\} = \left\{0, 0, 4\frac{19}{20}, 6\frac{1}{12}, 2\frac{3}{5}, 0\right\} \geq 0,\ j = 1,\ldots,6.$$

Thus, according to the optimality feasible condition, no more variable can be found to enter the basis, and the OS for the problem (9) is;

$$\left\{\tilde{z}_1; X_1(x_1, x_2, x_3)\right\} = \left\{(50,23,50); X_1(4,5,0)\right\}.$$

Using the solution algorithm in Section 6, one can find the OSs for other $\tilde{z}_i; i = 2,\ldots,4$ as shown in Table V.

$X_i\ (x_1, x_2, x_3)$

Now, by utilizing (7), the result is a single FLP problem as follows:

TABLE IV
THE STATUS OF THE SOLUTION-3

B	$\tilde{ci}$	RRHS	(5, 2, 7)	(6, 3, 6)	(5, 3, 7)	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	Min ratio
			x_1	x_2	x_3	s_1	s_2	s_3	
x_1	(5,2,5)	4	1	0	$\frac{6}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	0	
x_2	(6,3,6)	5	0	1	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{3}{10}$	0	
s_3	$\tilde{0}$	11	0	0	11	1	$-\frac{1}{2}$	1	
	$\tilde{z}$	(50, 23, 50)	$\tilde{0}$	$\tilde{0}$	$\left(\frac{23}{5},\frac{56}{5},\frac{63}{5}\right)$	$\left(\frac{16}{3},\frac{7}{3},\frac{16}{3}\right)$	$\left(\frac{23}{10},\frac{11}{10},\frac{23}{10}\right)$	$\tilde{0}$	$\Re$

TABLE V
THE STATUS OF THE OBJECTIVE FUNCTIONS

Objective function $(\tilde{Z}i)$	$\tilde{\upsilon}_i$	$X_i(x_1, x_2, x_3)$
$Max\ \tilde{Z}1$	(50, 23, 50)	X_1 (4, 5, 0)
$Max\ \tilde{Z}2$	(41, 53, 89)	X_2 (4, 5, 0)
$Max\ \tilde{Z}3$	(−19, 32, 13)	X_3 (4, 5, 0)
$Max\ \tilde{Z}4$	(−28, 36, 46)	X_4 (4, 5, 0)

$$Max\ \tilde{Z}(x) =_{\Re} \left[\frac{\{(5,2,5)x_1+(6,3,6)x_2+(5,3,7)x_3\}}{50,23,50}\right)\oplus$$
$$\{(4,7,11)x_1+(5,5,9)x_2+(3,6,10)x_3\}\oslash(41,53,89)]$$
$$\oplus\left[\frac{\left\{(-1,3,2)x_1+(-3,4,1)x_2+\left(\frac{-5}{2},9,7\right)x_3\right\}}{-19,32,13}\right)\Theta$$
$$\{(-3,4,4)x_1+(-3,4,6)x_2+(-3,4,8)x_3\}\oslash(-28,36,46)] \quad (10)$$
$$s.t. 3x_1 - x_2 + 3x_3 \le 7$$
$$-2x_1 + 4x_2 \le 12$$
$$-4x_1 + 3x_2 + 8x_3 \le 10$$
$$x_j \ge 0, j = 1,2,3$$

This is equivalent to:

$$Max\ \tilde{Z}(x) =_{\Re} \left(\frac{211}{5584},\frac{671}{673},\frac{99}{94}\right)x_1$$
$$+\left(\frac{-263}{11392},\frac{1653}{1336},\frac{746}{627}\right)x_2+\left(\frac{-568}{8665},\frac{312}{211},\frac{159}{113}\right)x_3$$
$$s.t. 3x_1 - x_2 + 3x_3 \le 7$$
$$-2x_1 + 4x_2 \le 12 \quad (11)$$
$$-4x_1 + 3x_2 + 8x_3 \le 10$$
$$x_j \ge 0, j = 1,2,3$$

The standard form of the above FLP problem is:

$$Max\ \tilde{Z}(x) =_{\Re} \left(\frac{211}{5584},\frac{671}{673},\frac{99}{94}\right)x_1+\left(\frac{-263}{11392},\frac{1653}{1336},\frac{746}{627}\right)x_2$$
$$+\left(\frac{-568}{8665},\frac{312}{211},\frac{159}{113}\right)x_3+\tilde{0}\sum_{j=1}^{3}s_j$$
$$s.t. 3x_1 - x_2 + 3x_3 + s_1 = 7$$
$$-2x_1 + 4x_2 + s_2 = 12$$
$$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$$
$$x_j \ge 0, j = 1,2,3 \quad (12)$$

Now, using $\Re$ in (2), through simplex method the solution of the FLP problem (12) is as shown in Table VI.

From Table VII, we have

$$\{(\tilde{z}_j - \tilde{c}_j)\} =_{\Re} \left\{\begin{array}{c}\left(\frac{-211}{5584},\frac{99}{94},\frac{671}{673}\right),\left(\frac{263}{11392},\frac{746}{627},\frac{1653}{1336}\right),\\ \left(\frac{568}{8665},\frac{159}{113},\frac{312}{211}\right),\tilde{0},\tilde{0},\tilde{0},\end{array}\right\}.$$
$$j = 1,\ldots,6$$

Since,

$$\{\gamma_j\} = \{\Re(\tilde{\gamma}_j)\} = \left\{-\frac{329}{6348},\frac{399}{11414},\frac{719}{8616},0,0,0\right\}, j = 1,\ldots,6$$

thus, x_1 should enter the basic solution and the leaving variable is s_1. The result is as shown in Table VII.

In Table VII, since

$$\{(\tilde{z}_j - \tilde{c}_j)\} =_{\Re} \left\{\begin{array}{c}\tilde{0},\left(\frac{81}{7721},\frac{396}{257},\frac{9661}{6155}\right),\left(\frac{161}{1558},\frac{351}{146},\frac{1033}{408}\right),\\ \left(\frac{61}{4843},\frac{671}{2019},\frac{33}{94}\right),\tilde{0},\tilde{0}; j = 1,\ldots,6\end{array}\right\}$$

and

$$\{\gamma_j\} = \{\Re(\tilde{\gamma}_j)\} = \left\{0,\frac{106}{5995},\frac{961}{7104},\frac{511}{29579},0,0\right\} \ge 0, j = 1,\ldots,6.$$

Thus, according to the optimality feasible condition, no more variable may enter the basis, and the OS for the problem (12) is

$$\{\tilde{Z}; X(x_1,x_2,x_3)\} = \left\{\left(\frac{1477}{16752},\frac{4697}{2019},\frac{231}{94}\right); X\left(\frac{7}{3},0,0\right)\right\}.$$

TABLE VI
THE STATUS OF THE COMPROMISE SOLUTION-1

B	$\tilde{ci}$	RHS	$\left(\frac{211}{5584},\frac{671}{673},\frac{99}{94}\right)$	$\left(-\frac{263}{11392},\frac{1653}{1336},\frac{746}{627}\right)$	$\left(-\frac{568}{8665},\frac{312}{211},\frac{159}{113}\right)$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	Min ratio
			x_1	x_2	x_3	s_1	s_2	s_3	
s_1	$\tilde{0}$	7	3	-1	3	1	0	0	$\frac{7}{3}\leftarrow$
s_2	$\tilde{0}$	12	-2	4	0	0	1	0	-
s_3	$\tilde{0}$	10	-4	3	8	0	0	1	-
	$\tilde{z}$	$\tilde{0}$	$\left(\frac{-211}{5584},\frac{99}{94},\frac{671}{673}\right)$	$\left(\frac{263}{11392},\frac{746}{627},\frac{1653}{1336}\right)$	$\left(\frac{568}{8665},\frac{159}{113},\frac{312}{211}\right)$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\Re$

TABLE VII
THE STATUS OF THE COMPROMISE SOLUTION-2

B	$\tilde{ci}$	RHS	x_1	x_2	x_3	s_1	s_2	s_3	Min ratio
x_1	$\left(\frac{211}{5584},\frac{671}{673},\frac{99}{94}\right)$	$\frac{7}{3}$	1	$\frac{-1}{3}$	1	$\frac{1}{3}$	0	0	
s_2	$\tilde{0}$		0	$\frac{10}{3}$	2	$\frac{2}{3}$	1	0	
s_3	$\tilde{0}$	$\frac{58}{3}$	0	$\frac{5}{3}$	12	$\frac{4}{3}$	0	1	
$\tilde{z}$	$\left(\frac{1477}{1672},\frac{4697}{2019},\frac{231}{94}\right)$	$\tilde{0}$	$\left(\frac{81}{7721},\frac{396}{257},\frac{9661}{6155}\right)$	$\left(\frac{161}{1558},\frac{351}{146},\frac{1033}{408}\right)$	$\left(\frac{61}{4843},\frac{671}{2019},\frac{33}{94}\right)$	$\tilde{0}$	$\tilde{0}$	$\Re$	

Moreover, this is the compromise solution for the original problem in (8).

VIII. CONCLUSION

We considered MFLP problems with BFS. We proposed a new technique to transform these multiple optimization problems into a single FLP problem. The compromise solution has been found for the resulted problem by using linear ranking function through simplex method. We believe the technique is practicable in real life.

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